Usually I'd prepare a review sheet before an exam like this, but the book already does a better job than I would have – I strongly recommend making sure you understand the *Review of the Key Ideas* at the end of each section.

The next page has a few practice problems you might find useful. A warning: I probably spent too long on computation problems in recitation, so you won't find any here. There will almost certainly be some on the exam, so do some problems from old tests or the recitation sheets!

Here are some computations you should be ready for (see recitation problems for examples):

- Decompositions: $A = LU$, $PA = LU$, $A = LDU$, $A = LDL^T$ (if A symmetric).
- Inverse of a square matrix.
- rref form of a matrix (and the various things you can read off of this – the rank, the nullspace, the pivot columns).
- All solutions to $A\mathbf{x} = \mathbf{0}$ (i.e., the nullspace of A; use the "special solutions" from free columns).
- All solutions to $A\mathbf{x} = \mathbf{b}$ (find a particular solution via elimination, putting 0s in free vars).

Math 18.06, r3/5
Problems #Review 1
March 3, 2013

1. Let A and B be the matrices

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}, \qquad B = \begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{pmatrix}.$$

   Does there exist a vector $\mathbf{b}$ such that $A\mathbf{x} = \mathbf{b}$ has a unique solution? Such that $B\mathbf{x} = \mathbf{b}$ has a unique solution?

2. Give an example of a matrix A such that

   - For some vector $\mathbf{b}$, $A\mathbf{x} = \mathbf{b}$ has no solutions.
   - For any $\mathbf{b}$ for which $A\mathbf{x} = \mathbf{b}$ has any solutions, it has infinitely many.

   What matrices have this property?

3. Suppose that an LU decomposition for A has $L = \begin{pmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$. What row operations must we have made when going from A to U by elimination?

4. Suppose that A is a 3×5 matrix of rank 2. If B is a 5×10 matrix, what are the possible ranks of AB? If C is 10×3, what are the possible ranks of CA?

5. Give an example of a matrix whose entries are all 0s or 1s, but which has a pivot that's something other than 1 or -1.

6. For which 3×3 matrices A does rref take the form $R = \begin{pmatrix} 1 & a & b \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$? (Describe conditions on the columns of A)

7. The inverse of an upper-triangular matrix is upper-triangular. Why?

8. (a) An upper triangular matrix is invertible if ________________________.
   (b) Let $\mathbf{v} = (1, 2, 3)$ be a three-dimensional vector. The set of 3×3 matrices with ($\mathbf{v}$ in the column space/$\mathbf{v}$ in the nullspace/both/neither) is a subspace of the space of 3×3 matrices.
   (c) The number of $n \times n$ permutation matrices with a "1" in the upper left is ___.

9. Suppose A is a 3×3 matrix which isn't invertible. Is it possible that there is 3×3 matrix B such that AB is invertible? Can AB be invertible if B has other dimensions?

10. Let V be the subspace of $\mathbb{R}^3$ consisting of vectors (x, y, z) for which $x + 2y + 3z = 0$. What is the dimension of V? Give a basis for V.

11. Let V be the vector space of polynomials of degree less than or equal to 2 (i.e. functions $f(x) = ax^2 + bx + c$). What is a basis for V? Do the polynomials with $f(1) = 0$ form a subspace? Those with $f(1) \geq 0$?