Today: 4.3, graphing rational functions

Reminder: - MML 4.2 due tonight!

- Quizzes start this week.

Math 121 (Lesieutre); §4.3; September 6, 2017

The steps for graphing a rational function:

1. Factor the numerator and denominator, and write the function in lowest terms.
2. Set the numerator equal to zero to find the x-intercepts (don't forget about multiplicity!)
3. Plug in $x = 0$ to find the y-intercept.
4. Set the denominator equal to zero to find the vertical asymptotes.
5. Find the horizontal asymptotes, depending on the degree of numerator and denominator:
   (a) If degree of numerator is less than degree of denominator, $y = 0$ is asymptote.
   (b) If degree of num. = degree of denom., then $y = \dfrac{\text{leading coef of numerator}}{\text{leading coef of denominator}}$ is asymptote.
   (c) If degree of numerator is greater than degree of denominator, there is no horizontal asymptote.
6. Split the x-axis into intervals, breaking it up whereever the numerator or denominator is 0. For each interval, figure out if the graph is above or below the axis in that interval, by plugging in a test number.

1. $f(x) = \dfrac{x^2 + x - 12}{x^2 - 4} = \dfrac{(x-3)(x+4)}{(x-2)(x+2)}$ 1.

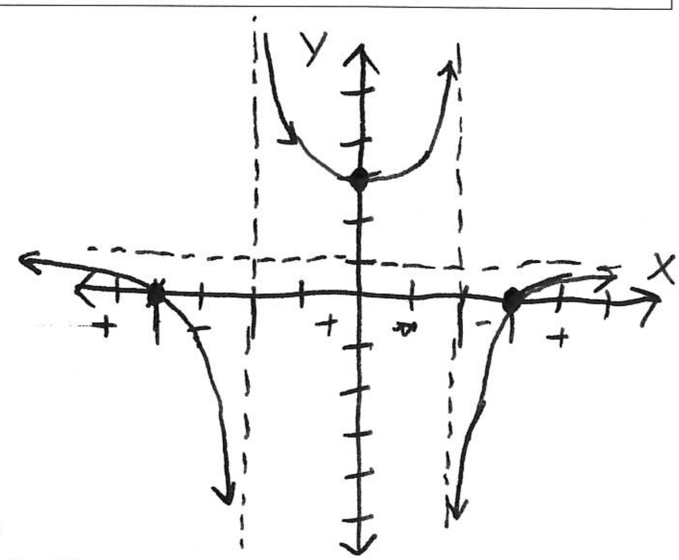
2. $x=3$, $x=-4$.

3. y-int $= 3$

4. vert asy: $x=2$, $x=-2$

5. b) $\Rightarrow$ horizon asy: $y = \frac{1}{1} = 1$.

6. num or den $=0$ at $x =$ ~~2, 2, 3, 4~~ $-4, -2, 2, 3$.



2. $g(x) = \dfrac{2x^2 + 2x - 24}{x^2 - x - 6}$ (there's a cancellation – so what?)

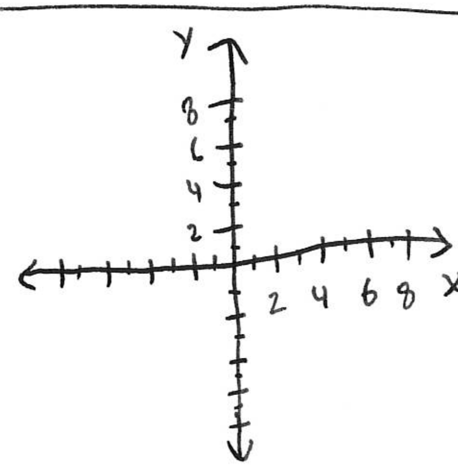
~~Six ranges~~ $(-\infty, -4)$ $f(-10) = \frac{13}{16}$

Five ranges: $(-4, -2)$ $f(-3) = -6/5$

$(-2, 2)$ $f(0) = 3$

in each range, $(2, 3)$ $f(5/2) = -13/9$

either above axis, or below. $(3, \infty)$ $f(4) = 2/3$.



~~3. $h(x) = \dfrac{x}{x^2 - 4}$~~

2. $\dfrac{2x^2+2x-24}{x^2-x-6} = \dfrac{2(x^2+x-12)}{x^2-x-6} = \dfrac{2(x-3)(x+4)}{(x-3)(x+2)} = \dfrac{2(x+4)}{x+2}$

$x \neq 3$
$x \neq -2$.
is domain.

use cancelled version for everything but domain!

Step 2: x-int: $x=-4$

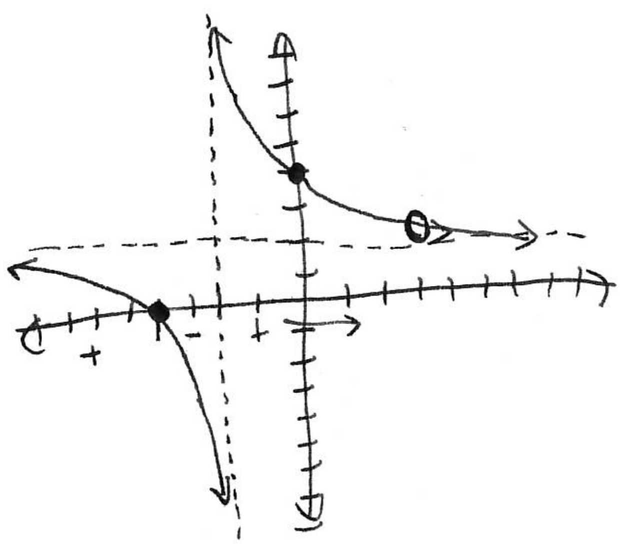
Step 3: $y=4$

Step 4: $x=-2$ is only vertical asy.

(not $x=3$)

Step 5: horizontal asy: $\frac{2}{1}=2$. $y=2$.

Step 6:



"cut points":

$x=-4$, $x=-2$

$(-\infty,-4)$: $f(-10)$ is $+$

$(-4,-2)$: $f(-3)$ is $-$

$(-2,\infty)$: $f(0)$ is $+$

But $x=3$ not in domain!

There's a "hole in the graph"

mark it with a "o".

$$\frac{2(x+4)}{x+2}.$$

4. $i(x) = \dfrac{x^2+3x+2}{x-1}$ (there's a slant asymptote – can you figure out what it is?)

To find slant asymptote:

Long division:

$$\begin{array}{r} x+4 \\ x-1 \overline{\big)\, x^2+3x+2} \\ \underline{x^2-x} \\ 4x+2 \\ \underline{4x-4} \\ 6 \end{array}$$

6 remandor.

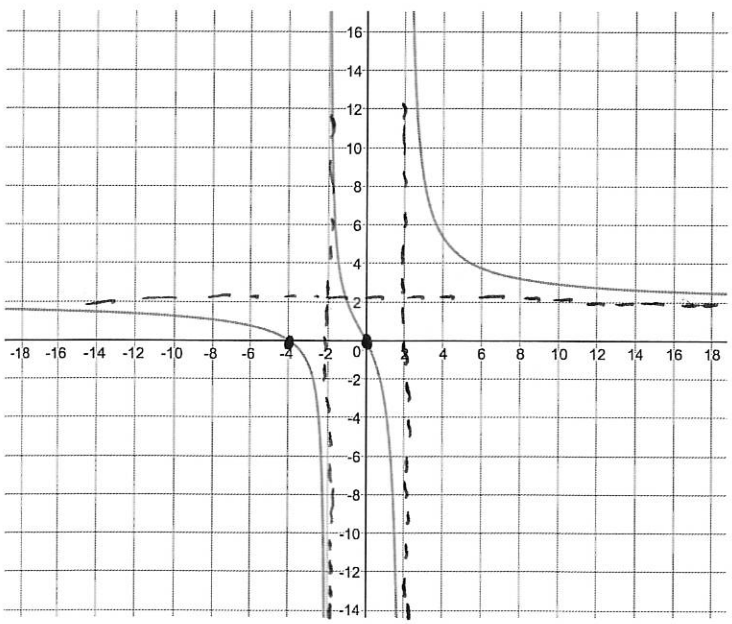
so...

$x^2+3x+2=(x-1)(x+4)+6$

$\dfrac{x^2+3x+2}{x-1} = \boxed{x+4} + \dfrac{6}{x-1}$. goes to 0 as $x \to \infty$.

$y=x+4$ is slant asy.

5. Given the graph below, find all asymptotes, intercepts, end behavior, domain, and range. Then use this to find a possible formula for the rational function.



x-intercepts: $x=-4$, $x=0$

(that's where numerator is 0)

y-intercept: $y=0$

vertical asy: $x=2$, $x=-2$

(that's where denominator is 0)

horizontal asy: $y=2$.

(same degree, $\dfrac{\text{leading of num}}{\text{leading of denomin}}=2$)

Guess: $\dfrac{2x(x+4)}{(x+2)(x-2)} = \dfrac{2x^2+8x}{x^2-4}$