Math 121 (Lesieutre); §4.2: Rational functions; September 1, 2017

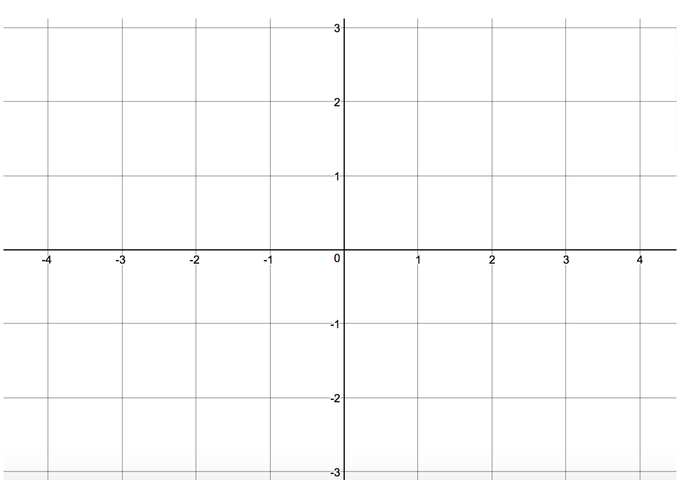
1. What is a rational function?

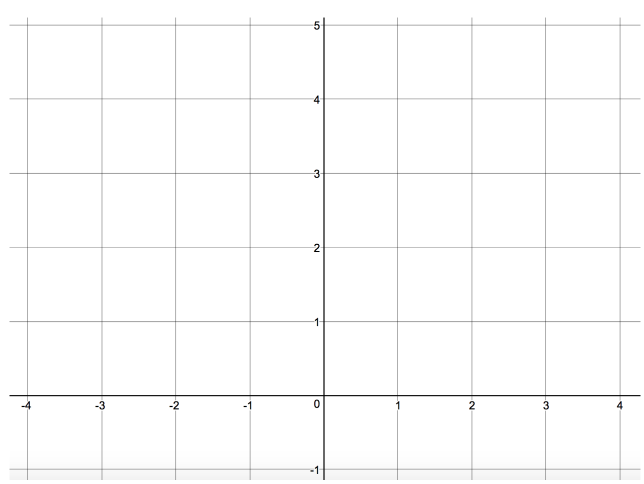
2. Find the domains of each of the following rational functions:

$$r_1(x) = \frac{2}{x-4}, \qquad r_2(x) = \frac{1}{x^2-25}, \qquad r_3(x) = \frac{5}{x^2+3}, \qquad r_4(x) = \frac{x^2-1}{x^2+3x+2}.$$

3. Sketch the graphs of these two important rational functions, using the graph paper below.

$$r_1(x) = \frac{1}{x}, \qquad r_2(x) = \frac{1}{x^2}.$$





4. How can you tell when a rational function has a vertical asymptote? Find the vertical asymptotes of the following rational functions:

$$r_1(x) = \frac{x}{x^2 - 7x + 12}, \qquad r_2(x) = \frac{x^2 - 9}{x^2 - 9x + 18}.$$

5. Finding Horizontal or Oblique asymptotes: These asymptotes describe the "long term" behavior of a function. What happens depends on how the degrees of the numerator and denominator compare...

a) Case 1: Degree of the numerator is less than the degree of the denominator: $r_1(x) = \frac{1}{x}$

b) Case 2: Degree of the numerator equals the degree of the denominator: $r_2(x) = \frac{2x^2+1}{3x^2+5}$

c) Case 3: Degree of the numerator is greater than the degree of the denominator: $r_3(x) = \frac{x^3+2}{x}$

d) Subcase 3': If the degree of the numerator is exactly one more than the degree of the denominator there is no horizontal asymptote, but you do get an Oblique (or Slant) asymptote.