Math 210 (Lesieutre)
11.7: Motion in space
January 25, 2017

Problem 1. Consider the path $\mathbf{r}(t) = \langle \cos t, \sin t, t^2 \rangle$. Compute the velocity, speed, and acceleration, all as functions of t.

Problem 2. A particle moves in a circular pattern, given by $\mathbf{r}(t) = \langle 2\cos t, 2\sin t \rangle$.

a) Compute the velocity, acceleration, and speed, as functions of t.

b) Sketch the path of the particle. For $t = \pi/4$, draw the vectors $\mathbf{r}(t)$, $\mathbf{v}(t)$, and $\mathbf{a}(t)$. Do these seem to make physical sense?

Problem 3. Suppose that a particle moves in a straight-line path $\mathbf{r}(t) = \mathbf{r}_0 + \mathbf{v}t$. What are the velocity and acceleration?

Problem 4. A particle has acceleration $\mathbf{a}(t) = \langle 2t, -1 \rangle$. Suppose that at time 0, it has position $\mathbf{r}(0) = \langle 2, 3 \rangle$ and velocity $\mathbf{v}(t) = \langle 0, 1 \rangle$. Find $\mathbf{r}(t)$.

Problem 5. A batter hits a baseball with initial velocity $\langle 100, 100, 100 \rangle$ (a pop-up down the right field line; let's say the units are ft/sec).

a) What is $\mathbf{a}(t)$? Assume the only force acting on the ball is gravity.

b) Solve for $\mathbf{v}(t)$ and $\mathbf{r}(t)$.

c) At what time t does the ball hit the ground?

Problem 6. a) What is

$$\frac{d}{dt} |\mathbf{x}(t)|^2,$$

in terms of $\mathbf{x}(t)$ and its derivatives?

b) Suppose that $\mathbf{x}(t)$ has constant length. What does this tell you about the relationship between $\mathbf{x}(t)$ and $\mathbf{x}'(t)$?