

Math 210 (Lesieutre)

13.5: Triple integrals in spherical coordinates

March 17, 2017

**Problem 1.** a) Find the spherical coordinates for the point  $(x, y, z) = (0, 1, 1)$ .

b) Find the rectangular coordinates for the point  $(1, \pi/4, \pi/4)$ .

**Problem 2.** Use a triple integral to compute the volume of the unit sphere.

**Problem 3.** You want to integrate the function  $x^2 + y^2 + z^2$  over the portion of the earth with latitude greater than  $45^\circ$  N. Convert this to an integral in spherical coordinates. (Assume the radius of the earth is 4000.)

**Problem 4.** a) Use a triple integral to compute the volume of the top half of the unit hemisphere.

b) Find the integral of the function  $z$  over the top half of the hemisphere.

c) To find the  $z$ -coordinate of the center of mass of a region in 3D, you can use the formula

$$z_{cm} = \frac{\iiint_R z \, dV}{\iiint_R 1 \, dV}.$$

What is the center of mass of the northern hemisphere of the unit sphere?