Math 210 (Lesieutre)
Change of variables in multiple integrals
March 27, 2017

Problem 1. Consider the transformation T given by $x = 2u$, $y = 4u + v$.

a) Let S be the region in uv-plane given by the unit square. To what region R in the xy-plane does T send S?

b) What is the Jacobian for the transformation T?

c) Compute $\iint_R \sqrt{2x(y-2x)}\, dA$.

Problem 2. Consider the transformation T given by $x = u\cos v$, $y = u\sin v$.

a) Let R be the top half of a circle of radius 2, in the xy-plane. What region in the uv-plane is mapped to R by the transformation T?

b) What is the Jacobian for the transformation T?

c) Compute $\iint_R e^{-x^2-y^2}\,dA$

Problem 3. Make a substitution to evaluate the integral of $\sqrt{\frac{x+y}{x-y}}$ over a square R with vertices at $(2,0)$, $(3,1)$, $(3,-1)$, and $(4,0)$.

Problem 4. Let R be the region bounded by $x=-1$, $x=1$, $y=x^2$, and $y=x^2+1$. Compute the area by integrating the function 1 over the region, using a substitution.