Math 210 (Lesieutre)
14.1: Vector fields
March 29, 2017

Problem 1. Sketch each of the following vector fields.

a) $\mathbf{F}_1(x, y) = \langle x, y \rangle$

b) $\mathbf{F}_2(x, y) = \langle 3, 0 \rangle$

c) $\mathbf{F}_3(x, y) = y\mathbf{i} - x\mathbf{j}$

d) $\mathbf{F}_4(x, y) =$ student's choice (make one up, not constant)

Problem 2. Write down formulas for the vector fields described.

a) A field $\mathbf{F}$ which always points clockwise, and has length 1 for any x and y.

b) A 3D vector field which points directly in towards the origin, with length proportional to $1/r^2$ (this could be a gravitational field).

Problem 3. Suppose that a mass rests at $(0,0)$. The gravitational potential due to the mass at a point (x,y) is given by $f(x,y) = \frac{1}{r}$, where r is the distance from (x,y) to $(0,0)$.

a) Compute the gradient field associated to this potential function.

b) Try to sketch your vector field. Do the vectors get shorter or longer the further we get from the origin?

Problem 4. Compute and sketch the gradient field associated with the function $f(x,y) = \tan^{-1}(y/x)$.

Problem 5. Consider the two fields

$$\mathbf{F}_1(x,y) = \left\langle x^2, y^2 \right\rangle, \quad \mathbf{F}_2(x,y) = \left\langle y^2, x^2 \right\rangle.$$

One of these is the gradient field of some function, and the other one isn't. Which is which? How can you tell?