

Math 210 (Lesieutre)

14.3: Conservative vector fields, I

April 3, 2017

Problem 1. Consider the two fields

$$\mathbf{F}_1(x, y) = \langle x^2, y^2 \rangle, \quad \mathbf{F}_2(x, y) = \langle y^2, x^2 \rangle.$$

One of these is a conservative field, and the other one isn't. Which is which? Find the potential function.

Problem 2. Check whether the field $\mathbf{F} = \langle \sin y, x \cos y + 1 \rangle$ is conservative. If it is, find a potential function.

Problem 3. Consider the vector field

$$\mathbf{F}(x, y, z) = \langle yz, xz + ze^{yz}, xy + ye^{yz} + \sin z \rangle.$$

This field is conservative. Find a potential function.

Problem 4. Consider the field $\mathbf{F}_1 = \langle x^2, y^2 \rangle$ from the first problem. Compute the line integral $\int_C \mathbf{F}_1 \cdot d\mathbf{r}$ for two paths from $(0, 0)$ to $(1, 1)$: first, a straight line path. Second, a path that goes from $(0, 0)$ to $(1, 0)$ in a straight line, and then from $(1, 0)$ to $(1, 1)$ in a straight line.