Math 210 (Lesieutre)
14.4: Green's theorem, I
April 7, 2017

Circulation form:	Flux form:
$\oint_C \mathbf{F} \cdot d\mathbf{r} = \iint_R \operatorname{curl} \mathbf{F} \, dA$	$\oint_C \mathbf{F} \cdot \mathbf{n} \, ds = \iint_R \operatorname{div} \mathbf{F} \, dA$
$\operatorname{curl} \mathbf{F} = \frac{\partial g}{\partial x} - \frac{\partial f}{\partial y}$	$\operatorname{div} \mathbf{F} = \frac{\partial f}{\partial x} + \frac{\partial g}{\partial y}$

Problem 1. Compute $\operatorname{div} \mathbf{F}$ and $\operatorname{curl} \mathbf{F}$ for the indicated vector fields.

a) $\mathbf{F}_1(x, y) = \langle x, y \rangle$

b) $\mathbf{F}_2(x, y) = \langle -y, x \rangle$

Problem 2. Let $\mathbf{F} = \langle -y, x \rangle$, and let C be a path around the unit circle starting at $(1, 0)$ and going counterclockwise. Verify the circulation form of Green's theorem by computing both sides directly.

Problem 3. Let R be the region bounded by $y = 1 - x^2$ and $y = 0$, and let C be a path that goes around the region counterclockwise, starting at $(1, 0)$. Verify the flux form of Green's theorem for the vector field $\mathbf{F} = \langle x + y, xy \rangle$.

Problem 4. What does the circulation form of Green's theorem tell us when $\mathbf{F}$ is a conservative vector field?