Math 210 (Lesieutre)
14.4: Green's theorem, II
April 10, 2017

Circulation form:	Flux form:
$\oint_C \mathbf{F} \cdot d\mathbf{r} = \iint_R \operatorname{curl} \mathbf{F} \, dA$	$\oint_C \mathbf{F} \cdot \mathbf{n} \, ds = \iint_R \operatorname{div} \mathbf{F} \, dA$
$\operatorname{curl} \mathbf{F} = \frac{\partial g}{\partial x} - \frac{\partial f}{\partial y}$	$\operatorname{div} \mathbf{F} = \frac{\partial f}{\partial x} + \frac{\partial g}{\partial y}$

Problem 1. Let $\mathbf{F} = \langle -y, x \rangle$, and let C be a path around the unit circle starting at $(1, 0)$ and going counterclockwise. Verify the circulation form of Green's theorem by computing both sides directly.

Problem 2. Suppose that you have a region R bounded by a curve C. What does the circulation form of Green's theorem tell you when it's applied to the field $\mathbf{F} = \langle 0, x \rangle$?

Problem 3. Use Green's theorem to compute the outward flux of the field $\mathbf{F} = \langle x^2 y, y \rangle$ across a counterclockwise semicircular path from $(2, 0)$ to $(0, -2)$.

Problem 4. Evaluate the line integral $\oint_C (2x + y)\, dx + (x + y)\, dy$, where C is a square with opposite corners at $(0, 0)$ and $(1, 1)$.

Problem 5. Let R be the unit circle. Convert the double integral $\iint_R xy + \sin x \, dA$ into a line integral; you do not need to evaluate.