Math 210 (Lesieutre)
14.5: Divergence and curl
April 12, 2017

Problem 1. Let $\mathbf{F}$ be the vector field $\mathbf{F} = \langle xyz, ye^x, z \rangle$.

a) Compute the divergence $\nabla \cdot \mathbf{F}$.

b) Compute the curl $\nabla \times \mathbf{F}$.

c) Compute the divergence of the curl, $\nabla \cdot (\nabla \times \mathbf{F})$.

Problem 2. Suppose that we have a 3D vector field of the form $\mathbf{F} = \langle f(x, y), g(x, y), 0 \rangle$ (i.e. f and g only depend on x and y, and $h = 0$). What is the (3D) curl $\nabla \times \mathbf{F}$? What do you notice about this?

Problem 3. Consider the function $\phi(x, y, z) = xy^2z$.

a) Compute the gradient $\mathbf{F} = \nabla\phi$. (This $\mathbf{F}$ is a conservative field.)

b) Compute the curl of the gradient, $\nabla \times \nabla\phi$.

Problem 4. Consider the vector field $\mathbf{F} = \langle 1, 0, 3\rangle \times \mathbf{r}$, where $\mathbf{r} = \langle x, y, z\rangle$. This is an example of a *rotation vector field*.

a) Multiply out the cross product to find a formula for $\mathbf{F}$.

b) Compute the curl of this field.