

Math 210 (Lesieutre)
14.6: Surface integrals
April 15, 2017

Problem 1. Let S be the outer shell of a cylinder of height 3 and radius 4, with base centered at the origin.

a) Give a parametrization for S , and compute $\mathbf{t}_u \times \mathbf{t}_v$.

b) How would you compute the integral $\iint_S xz \, dS$?

Problem 2. Give parametrizations for the following regions.

a) A sphere of radius 7.

b) The part of the plane $z = 5 + 2x + y$ lying above the unit square in the xy -plane.

Problem 3. Consider the surface S comprising the portion of the paraboloid $z = 1 - x^2 - y^2$ lying above the unit disk.

a) Give a parametrization of S ; be sure to include bounds on u and v .

b) Compute the tangent vectors $\mathbf{t}_u = \frac{\partial \mathbf{r}}{\partial u}$, $\mathbf{t}_v = \frac{\partial \mathbf{r}}{\partial v}$, the cross product $\mathbf{t}_u \times \mathbf{t}_v$, and the length $|\mathbf{t}_u \times \mathbf{t}_v|$.

c) Set up an integral for the surface area of this part of the paraboloid. If you have time, compute the integral.

Problem 4. Let S be the top half of a sphere of radius 2 centered at the origin.

a) Give a parametrization of S ; be sure to include bounds on u and v .

b) Compute the tangent vectors $\mathbf{t}_u = \frac{\partial \mathbf{r}}{\partial u}$ and $\mathbf{t}_v = \frac{\partial \mathbf{r}}{\partial v}$.

c) Compute the surface area of this portion of the sphere. Does your answer match the one you have learned before?

d) Compute the average value of the angle ϕ on this part of the sphere. What is the average latitude of a point in the northern hemisphere?