

Math 210 (Lesieutre)
Quiz 3
February 8, 2017

Name: _____

Problem 1. Compute the limit $\lim_{(x,y) \rightarrow (0,0)} \frac{xy}{x^2+y^2}$ along the path $y = mx$.

Does the limit $\lim_{(x,y) \rightarrow (0,0)} \frac{xy}{x^2+y^2}$ exist?

We have

$$\begin{aligned} \lim_{\substack{(x,y) \rightarrow (0,0) \\ \text{along } y = mx}} \frac{xy}{x^2 + y^2} &= \lim_{x \rightarrow 0} \frac{x(mx)}{x^2 + (mx)^2} = \lim_{x \rightarrow 0} \frac{mx^2}{(1 + m^2)x^2} \\ &= \lim_{x \rightarrow 0} \frac{m}{1 + m^2} = \frac{m}{1 + m^2}. \end{aligned}$$

This depends on m . That means that we get different limits along different paths, which means that $\lim_{(x,y) \rightarrow (0,0)} \frac{xy}{x^2+y^2}$ does not exist, according to the two-path test.