

Math 210 (Lesieutre)
Quiz 8
March 29, 2017

Name: _____

Problem 1. Let T be the transformation defined by

$$\begin{aligned}x(u, v) &= u^2 + v \\y(u, v) &= v^2 + u\end{aligned}$$

Suppose that T sends a region S in the uv -plane to R in the xy -plane.

You want to compute

$$\iint_R xy \, dx \, dy.$$

Make a substitution to convert this into an integral in terms of u and v . Your answer should be a double integral over S ; don't worry about specifically what the region is.

The key thing to remember is that we need to know the Jacobian. This is given by

$$J(u, v) = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix} = \begin{vmatrix} 2u & 1 \\ 1 & 2v \end{vmatrix} = 4uv - 1.$$

The function to integrate becomes $xy = (u^2 + v)(v^2 + u)$. So our answer is

$$\iint_S (u^2 + v)(v^2 + u)(4uv - 1) \, du \, dv.$$