Math 210 (Lesieutre)
11.1: Vectors in the plane
January 9, 2017

Problem 1. *Let* $\mathbf{u} = \langle 1, 3 \rangle$ *and* $\mathbf{v} = \langle 0, -2 \rangle$.

a) *Compute* $\mathbf{u} + 2\mathbf{v}$*, both geometrically and algebraically. Do your answers match?*
We have

$$\mathbf{u} + \mathbf{v} = \langle 1, 3 \rangle + 2 \langle 0, -2 \rangle = \langle 1, 3 \rangle + \langle 0, -4 \rangle = \langle 1, -1 \rangle .$$

b) *Compute* $\mathbf{u} - \mathbf{v}$*, both geometrically and algebraically.*
For this one,

$$\mathbf{u} - \mathbf{v} = \langle 1, 3 \rangle - \langle 0, -2 \rangle = \langle 1, 5 \rangle .$$

c) *Compute* $3\mathbf{u}$*, both geometrically and algebraically.*
Multiplying both components by 3, we obtain

$$3\mathbf{u} = \langle 3, 9 \rangle .$$

d) *What is the magnitude of* $\mathbf{v}$ *(i.e.* $|\mathbf{v}|$*?) Does the formula match the picture?*
For this one, the formula gives $|\mathbf{v}| = \sqrt{0^2 + (-2)^2} = \sqrt{4} = 2$. This obviously matches the length we get if we draw the vector, which points straight towards the bottom of the page and has length 2.

Problem 2. a) *What is the vector pointing from* $(1, 1)$ *to* $(4, -3)$*?*
Again, try to do this one by drawing the picture. We want the vector

$$\mathbf{v} = \langle 4, -3 \rangle - \langle 1, 1 \rangle = \langle 3, -4 \rangle .$$

b) *Find a unit vector parallel to the vector in your answer from (a).*
We need

$$\mathbf{u} = \frac{\mathbf{v}}{|\mathbf{v}|} = \frac{\langle 3, -4 \rangle}{\sqrt{3^2 + (-4)^2}} = \frac{\langle 3, -4 \rangle}{5} = \left\langle \frac{3}{5}, -\frac{4}{5} \right\rangle .$$

c) *Find a vector with length* 7 *parallel to the vector in your answer from (a).*
We want 7 times our answer from (b), which is

$$\mathbf{w} = \left\langle \frac{21}{5}, -\frac{28}{5} \right\rangle .$$

Problem 3. *Relative to the air, an airplane is flying 30 degrees west of north, with speed* 500 *MPH. The wind is traveling due north at* 100 *MPH. What is the velocity vector of the airplane relative to the ground?*

First we need to translate to find the components of the airplane vector. In polar coordinates, our vector has angle $90 + 30 = 120$. The x-component is $500 \cos 120 = 500(-1/2) = -250$. The y-component is $500 \sin 120 = 500(\sqrt{3}/2) = 250\sqrt{3}$. So the velocity relative to the air is $\left\langle -250, 250\sqrt{3} \right\rangle$. The velocity of the air relative to the ground is $\langle 0, 100 \rangle$.

Let me write $\mathbf{v}_{X/Y}$ for the speed of X relative to Y. As we saw in class,

$$\mathbf{v}_{\text{plane/ground}} = \mathbf{v}_{\text{plane/air}} + \mathbf{v}_{\text{air/ground}} = \left\langle -250, 250\sqrt{3} \right\rangle + \langle 0, 100 \rangle = \left\langle -250, 250\sqrt{3} - 100 \right\rangle.$$

Problem 4. *A* 10-*pound weight is suspended from two strings, each making a* 45 *degree angle with the ceiling. How much force is exerted on the mass by each of the strings?*

This is a classic sort of problem. Since the situation is symmetric, the two forces from the strings are equal in magnitude, but in different directions. Let's say M is the answer. The two force vectors from the strings are

$$\mathbf{F}_1 = \left\langle -M\frac{\sqrt{2}}{2}, M\frac{\sqrt{2}}{2} \right\rangle$$
$$\mathbf{F}_2 = \left\langle M\frac{\sqrt{2}}{2}, M\frac{\sqrt{2}}{2} \right\rangle$$

(the first of those is $M \cos 45°$ for the left string, etc; similar to the previous problem)

The gravitational force is $\mathbf{g} = \langle 0, -10 \rangle$: a force of 10 directly downward.

Adding these up we should get 0: if the object isn't moving, the forces have to balance out:

$$\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{g} = \mathbf{0}.$$

So $\left\langle 0, \sqrt{2}M \right\rangle + \langle 0, -10 \rangle = \langle 0, 0 \rangle$, which means $\sqrt{2}M = 10$, so $M = 10/\sqrt{2} = 5\sqrt{2}$. (This tells use how strong each string needs to be to keep the mass from falling: each individually is holding the same force as if it were supporting a single $5\sqrt{2} \approx 7.07$.