Math 210 (Lesieutre)
12.1: Planes and surfaces, part 2
February 1, 2017

Problem 1. *Consider the two planes given by the following equations:*

$$x + 2y + 3z = 0$$
$$x - y + z = 2$$

These planes intersect in a line. Give a parametrization of this line, and check that your line is actually contained in the first plane.

First we need to find a point on the line. There are many points, and you can solve for one however you want. One way is just plug in $z = 0$ and then figure out what x and y have to be. You can also just guess one, which is what I am about to do: $(3, 0, -1)$.

Now we need to find the direction vector. How to do that? Well, it's perpendicular to both of the normal vectors, which are $\langle 1, 2, 3\rangle$ and $\langle 1, -1, 1\rangle$. So we can use the cross product $\mathbf{v} = \langle 1, 2, 3\rangle \times \langle 1, -1, 1\rangle$ as our normal vector! This guy is $\mathbf{v} = \langle 5, 2, -3\rangle$.

So our line is

$$\mathbf{r}(t) = \mathbf{r}_0 + t\mathbf{v} = \langle 3, 0, -1\rangle + t\langle 5, 2, -3\rangle = \langle 3 + 5t, 2t, -1 - 3t\rangle .$$

To see whether it indeed line in the plane, just plug this in to the equation for the plane. You get $x + 2y + 3z = (3 + 5t) + 2(2t) + 3(-1 - 3t) = 0$, so it works as it should. You could check the same thing with the second plane if you were so inclined.

Problem 2. a) *Graph the cylinder $x^2 + 2x + 4z^2 = 15$.*

This doesn't involve y, so we should try to graph the x and z part and then "draw the walls" by including all possible y-values.

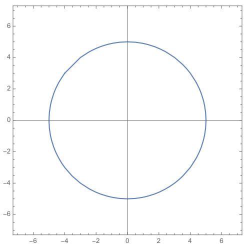
First, complete the square to obtain $(x - 1)^2 + 4z^2 = 16$, or perhaps easier, $((x - 1)/4)^2 + (z/z)^2 = 1$ This you should recognize: it's an ellipse. It looks like this:

Problem 3. *Consider the surface given by*

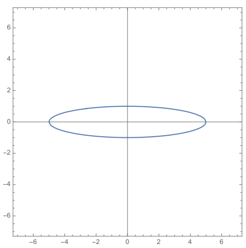
$$\frac{x^2}{25} + \frac{y^2}{25} + z^2 = 1.$$

a) *Sketch the xy-, xz-, and yz- traces of this figure. Use these to guide a drawing of the entire surface.*

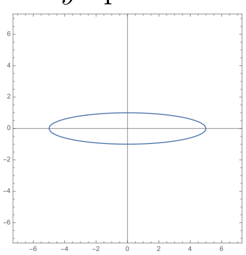
xy-plane:



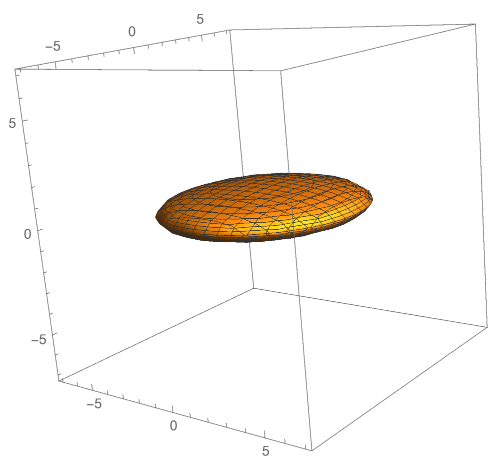
xz-plane:



yz-plane:



Putting it all together, we get the following figure. I'm drawing a "back view" that makes it easier to see the axes.



b) *Now consider the line* $\mathbf{r}(t) = \langle 3t, 4t, 1-t\rangle$. *At what points does the line intersect the surface?*

We do the same procedure we used on Monday: plug in $x(t) = 3 - 3t$, $y(t) = 4 - 4t$, and

$z(t) = 5t$ to our equation and solve for t. This gives

$$\begin{aligned}
\frac{x^2}{25} + \frac{y^2}{25} + z^2 &= 1 \\
x^2 + y^2 + 25z^2 &= 25 \\
(3-3t)^2 + (4-4t)^2 + 25(1-t)^2 &= 25 \\
25 - 50t + 50t^2 &= 25 \\
-50t + 50t^2 &= 0,
\end{aligned}$$

which means either $t = 0$ or $t = 1$. The points are then $\mathbf{r}(0) = \langle 0, 0, 1\rangle$ and $\mathbf{r}(1) = \langle 3, 4, 0\rangle$.

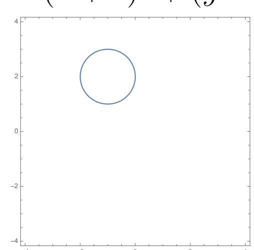
Problem 4. *Consider the figure* $x^2 + 2x + y^2 - 4y - z^2 + 4 = 0$.

a) *First, complete the square to simplify the equation for the surface.*

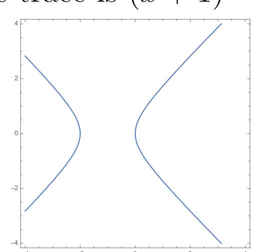
$$\begin{aligned}
x^2 + 2x + y^2 - 4y - z^2 &= -4 \\
x^2 + 2x + \mathbf{1} + y^2 - 4y + \mathbf{4} - z^2 &= \mathbf{5} - 4 \\
(x+1)^2 + (y-2)^2 - z^2 &= 1
\end{aligned}$$

b) *Next, sketch some traces of the figure. You can use the coordinate planes, but it might be better to use planes parallel to coordinate planes (for example, $y = 2$ might be a good one)*

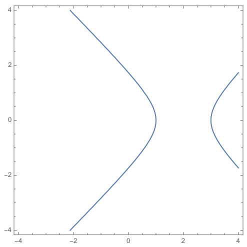
The plane $z = 0$. The trace is $(x+1)^2 + (y-2)^2 = 1$, a circle of radius 1.



The plane $y = 2$. The trace is $(x+1)^2 - z^2 = 1$, a hyperbola.



the plane $x = -1$:



c) *Use your traces to sketch the 3D surface.*

Putting it all together, we get the following figure. I'm drawing a "back view" that makes it easier to see the axes.