first, some leftovers from §1.9.

VOCAB

Suppose $T: \mathbb{R}^n \to \mathbb{R}^m$ is a linear transformation.

→ T is called onto (surjective) if every $\vec{b} \in \mathbb{R}^m$ is the image of some $\vec{x}$.

→ T is called one-to-one (injective) if every $\vec{b} \in \mathbb{R}^m$ is the image of at most one $\vec{x}$.

What these mean and how to check them.

Suppose T is given by a matrix A.

Onto. - means that $A\vec{x} = \vec{b}$ has a solution for every $\vec{b}$. (maybe infinitely many, maybe just one.)

- which means every $\vec{b}$ is a linear combination of the columns of A, i.e. the columns span $\mathbb{R}^m$.

- to check: run rref on A. the columns of A span $\mathbb{R}^m$ if there is a pivot in every row.

(a fact from §1.4 which John didn't say)

<u>one-to-one</u> -means $A\vec{x}=\vec{b}$ has at most one solution: either 0 or 1, but not infinitely many.

- might as well check for $\vec{b}=\vec{0}$: are there infinitely many solutions to $A\vec{x}=\vec{0}$?

- in other words, are the columns of A linearly independent?

- to check: use row reduction to solve $A\vec{x}=\vec{0}$. is there a free variable? if yes, infinitely many sols, so <u>not</u> one-to-one.

<u>Example</u>: $A=\begin{pmatrix} 1 & 2 & -1 & 4 \\ 0 & 1 & 3 & 5 \\ 0 & 0 & 0 & 1 \end{pmatrix}$. already in echelon form!

onto? yes! pivot in every row.

one-to-one? no! 4 columns in $\mathbb{R}^3$. can't be independent. (x_3 is free when we solve $A\vec{x}=\vec{0}$)

§1.10 More applications.

1. Some simple circuits.

a battery and some resistors. let's find the current through each part.

when current passes across a resistor, there's a "voltage drop" $V = RI$. (Ohm's law). some potential is used up.

<u>Kirchhoff's voltage law</u>: if we add up voltage drops around a loop equal to sum of voltage sources in the loop.

in example, top loop

voltage drop $3I_1 + 6I_1 - 6I_2 = 0$ no battery

(the middle resistor carries currents from both loops.)

bottom loop:

$1I_2 + 6I_2 - 6I_1 = 2$ battery

let's solve.

$$\left(\begin{array}{cc|c} 9 & -6 & 0 \\ -6 & 7 & 2 \end{array}\right) \to \left(\begin{array}{cc|c} 3 & -2 & 0 \\ -6 & 7 & 2 \end{array}\right) \to \left(\begin{array}{cc|c} 3 & -2 & 0 \\ 0 & 3 & 2 \end{array}\right)$$

$$\to \left(\begin{array}{cc|c} 3 & -2 & 0 \\ 0 & 1 & 2/3 \end{array}\right) \to \left(\begin{array}{cc|c} 3 & 0 & 4/3 \\ 0 & 1 & 2/3 \end{array}\right) \to \left(\begin{array}{cc|c} 1 & 0 & 4/9 \\ 0 & 1 & 2/3 \end{array}\right).$$

$$I_1 = 4/9, \quad I_2 = 2/3$$

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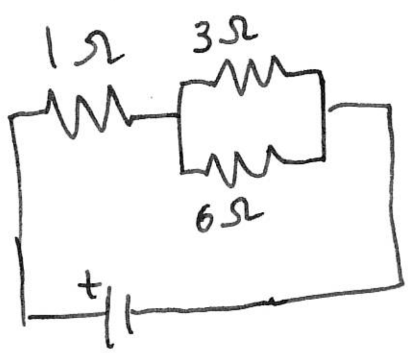
does this agree with physics classes?

same circuit:



resistors in parallel;
effective resistance is

$$\frac{1}{\frac{1}{3}+\frac{1}{6}} = \frac{1}{1/2} = 2\,\Omega.$$

in series with $1\,\Omega$:

$$1\,\Omega + 2\,\Omega = 3\,\Omega.$$

current in loop. $V = IR$

$$I = V/R = 2/3\ A. \checkmark$$

<u>Difference equations</u>

- Suppose every year, 5% of Chicagoans move to the suburbs, and 10% of suburbians move to the city.

- Suppose that in year 0 (=2015), there are 3 million people in city limits, 6 million outside.

- how many live in-city in 2016? (year 1) it's given by

$$\begin{pmatrix} 0.95 & 0.1 \\ 0.05 & 0.9 \end{pmatrix} \begin{array}{c} \begin{pmatrix} 3 \\ 6 \end{pmatrix} \end{array} \begin{array}{l} \leftarrow \text{city} \\ \leftarrow \text{suburb} \end{array} = \begin{pmatrix} 3.45 \\ 5.55 \end{pmatrix}.$$

in year 2, it's

$$\begin{pmatrix} 0.95 & 1 \\ 0.05 & 0.9 \end{pmatrix} \begin{pmatrix} 3.45 \\ 5.55 \end{pmatrix}$$

in general, $\vec{x}_{k+1} = A\vec{x}_k$. this is an example of a <u>difference equation</u>, and A is the migration matrix. more on this later in the course!