- Exam 1 will be returned Friday - sorry!

- Today: one more thing in 3.3, then start Ch. 4.

Inverses using Cramer's rule.

last time:

$$(i,j)\text{-entry of } A^{-1} = \frac{\det(A_i(\vec{e}_j))}{\det A}$$

replace i^{th} col of A with $\vec{e}_j$.

we can simplify this a little more:

i^{th} column has only one nonzero entry. Cofactor expansion down i^{th} column gives:

$$\det(A_i(\vec{e}_j)) = (-1)^{i+j} \det A_{ji}$$

cross out j^{th} row, i^{th} column

$$= C_{ji}\text{, called a "cofactor".}$$

so:

$$A^{-1} = \frac{1}{\det A}\begin{pmatrix} C_{11} & C_{21} & C_{31} \\ C_{12} & C_{22} & C_{32} \\ C_{13} & C_{23} & C_{33} \end{pmatrix}$$

each C_{ij} is a det!

$$A = \begin{pmatrix} 1 & 1 & 7 \\ 1 & 2 & 8 \\ 0 & 3 & 9 \end{pmatrix}.$$

what's the (2,2)-entry of A^{-1}?

$$C_{22} = (-1)^4 \det\begin{pmatrix} 1 & 7 \\ 0 & 9 \end{pmatrix} = 9.$$

so (2,2)-entry is

$$\frac{C_{22}}{\det A} = \frac{9}{6} = \boxed{\frac{3}{2}}$$

§4.1.

A <u>vector space</u> is any collection of objects you can add, and multiply by scalars, in a way that obeys the "usual rules of arithmetic":

- $\vec{u} + \vec{v} = \vec{v} + \vec{u}$
- $c(\vec{u} + \vec{v}) = c\vec{u} + c\vec{v}$

(10 rules needed)

<u>Examples:</u>

1. vectors in $\mathbb{R}^n$
2. all quadratic polynomials in a variable x.
   $(ax^2 + bx + c)$.

   if we add two of those, we get another:

   $$(x^2+2x+3) + (-7x^2 + 3x - 4) = (-6x^2 + 5x - 1)$$

3. 2x3 matrices
4. all functions of one variable: $\sin(x)$, e^x, $2+x^2$, ...

What's the point?

a lot of the stuff we've been with vectors, works just as well with other vector spaces.

example: $\frac{d}{dx}$: Functions in one var $\to$ Functions in one var

taking derivative

is a linear transformation!

$$\frac{d}{dx}(f+g) = \frac{d}{dx}f + \frac{d}{dx}g.$$

$$\frac{d}{dx}(cf) = c \cdot \frac{d}{dx}f.$$

Q. is $\frac{d}{dx}$ a one-to-one transformation?

NO:

$\frac{d}{dx}f = x^2$ has many solutions:

$$\frac{x^3}{3}, \quad \frac{x^3}{3}+7, \quad \frac{x^3}{3}+c.$$

$$A \vec{x} = \vec{b}$$

derivative

function

another function.

for $\vec{b} = x^2$ there are many $\vec{x}$ that work.

Subspace

a subspace of a vector space is a collection of of objects such that - when you add 2, you get another
- when mult. by scalar, you get another.

1) vector space $= \mathbb{R}^3$

consider all vectors with $x_1 = x_2 = x_3$. is it a subspace?

e.g. $\begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix} + \begin{pmatrix} 3 \\ 3 \\ 3 \end{pmatrix} = \begin{pmatrix} 5 \\ 5 \\ 5 \end{pmatrix}$

$-1 \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix} = \begin{pmatrix} -2 \\ -2 \\ -2 \end{pmatrix}$ ✓

2) vector space $= \mathbb{R}^3$

consider all vectors with $x_1 = 1$. is this a subspace?

no: $\underbrace{\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}}_{\text{these are both in my collections}} = \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix}$ but this isn't.