Announcements

- Piazza up and running!
  (I hope)

- class capacity now 85
  (but full)

- #1.2.4 fixed in typed HW

- Will add some problems from 1.3; won't be on quiz.

- next HW will have a few more problems
  than this one did.

HW due W, quiz on W.

covers previous MWF; will post all by W.

Finding general solution from rref.

a variable not in a pivot column is called a "free variable".

General solution:

- free variables can take any value
  (plug in any number you want)
- other variables determined by the free ones,
  using eqns from rref.

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example:
rref: $\left(\begin{array}{ccc|c} 1 & 0 & 3 & 1 \\ 0 & 1 & -1 & 2 \end{array}\right)$ $\Rightarrow$ x, y are pivot variables / basic variable
z is free.

pivots

free variable

$x+3z=1$

$x=1-3z$

$y-z=2$

$$\begin{cases} x = 1-3z \\ y = 2+z \\ z = \text{free} \end{cases}$$

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to get a solution, plug in $z=10$.

then $x=1-3(10)=-29$

$y=2+(10)=12.$

$x=-29,\ y=12,\ z=10$ is a solution

3 equations, five variable

$$\left(\begin{array}{ccccc|c} 1 & 2 & 1 & 4 & 1 & 4 \\ 1 & 2 & 2 & 7 & -2 & -3 \\ 2 & 4 & -1 & -1 & 0 & 7 \end{array}\right) \xrightarrow{\text{add } -1\times\text{row } 1} \left(\begin{array}{ccccc|c} 1 & 2 & 1 & 4 & 1 & 4 \\ 0 & 0 & 1 & 3 & -3 & -7 \\ 2 & 4 & -1 & -1 & 0 & 7 \end{array}\right) \xrightarrow{\text{add } -2\times\text{first row}}$$

$$\left(\begin{array}{ccccc|c} 1 & 2 & 1 & 4 & 1 & 4 \\ 0 & 0 & 1 & 3 & -3 & -7 \\ 0 & 0 & -3 & -9 & -2 & -1 \end{array}\right) \xrightarrow{\text{add } 3\times\text{row } 1} \underbrace{\left(\begin{array}{ccccc|c} 1 & 2 & 1 & 4 & 1 & 4 \\ 0 & 0 & 1 & 3 & -3 & -7 \\ 0 & 0 & 0 & 0 & -11 & -22 \end{array}\right)}_{\text{echelon form}} \xrightarrow{\text{multiply by } -1/11}$$

leading entry one

$$\left(\begin{array}{ccccc|c} 1 & 2 & 1 & 4 & 1 & 4 \\ 0 & 0 & 1 & 3 & -3 & -7 \\ 0 & 0 & 0 & 0 & 1 & 2 \end{array}\right) \xrightarrow{\text{add } 3\times\text{row } 3} \left(\begin{array}{ccccc|c} 1 & 2 & 1 & 4 & 1 & 4 \\ 0 & 0 & 1 & 3 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & 2 \end{array}\right) \xrightarrow{\text{add } -1\times\text{row } 3}$$

$$\left(\begin{array}{ccccc|c} 1 & 2 & 1 & 4 & 0 & 2 \\ 0 & 0 & 1 & 3 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & 2 \end{array}\right) \xrightarrow{\text{add } -1\times\text{row } 2} \underbrace{\left(\begin{array}{ccccc|c} 1 & 2 & 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 3 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & 2 \end{array}\right)}_{\text{rref!}}$$

The linear system corresponding to rref is:

$$x_1 + 2x_2 + x_4 = 3$$
$$x_3 + 3x_4 = -1$$
$$x_5 = 2.$$

The free variables are? The basic variables are?

Basic: x_1, x_3, x_5 Free: x_2, x_4

The general solution is?

$$\begin{cases} x_1 = 3 - 2x_2 - x_4 \\ x_2 = \text{free} \\ x_3 = -1 - 3x_4 \\ x_4 = \text{free} \\ x_5 = 2 \end{cases}$$

e.g plug in $x_2 = 1$, $x_4 = 2$ — or anything else

$\Downarrow$

$x_1 = -1$
$x_3 = -7$
$x_5 = 2$

plug this into equations: it works!

how many solutions?

0, if rref has a row that looks like

$$\left(0\ 0\ 0\ 0\ 0 \mid 3\right) \quad \swarrow \text{anything not } 0$$

no solutions to system.

∞ solutions, if there's a free variable.

1 solution: if no free vars, and no row $(0\ 0\ 0\ 0 \mid b)$.

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the system is "consistent" if there's 1 or ∞ solutions.

not consistent if no sols.

How to check if consistent?

→ figure out rref. is there a row $(0\ 0\ 0\ 0 \mid b)$?

yes: not consistent

no: consistent

Vectors

a vector is a matrix with only one column.

$$\vec{u} = \begin{pmatrix} 3 \\ 1 \end{pmatrix} \quad \vec{v} = \begin{pmatrix} 5 \\ 1 \\ 7 \end{pmatrix} \quad \vec{w} = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix}$$

a vectors with n entries $\Longleftrightarrow$ "a vector in $\mathbb{R}^n$"

the "R" is in a funny font: $\mathbb{R}^n$

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if two vectors are the same size, we can add them.

e.g. $\begin{pmatrix} 1 \\ 2 \end{pmatrix} + \begin{pmatrix} 3 \\ 4 \end{pmatrix} = \begin{pmatrix} 4 \\ 6 \end{pmatrix}$

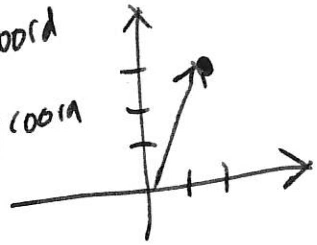
WARNING

- vector + vector (same size) is OK ✓
- vector + vector (different sizes) nonsense!
- number × vector OK ✓

  eg $5 \cdot \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 5 \\ 10 \end{pmatrix}$
- number + vector doesn't make sense. (usually)
- vector × vector usually doesn't make sense (except cross product for 3-dim'l vectors)

think of a vector as a point in 2D or 3D space.

$\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ ← x coord, ← y coord



similarly in 3D.

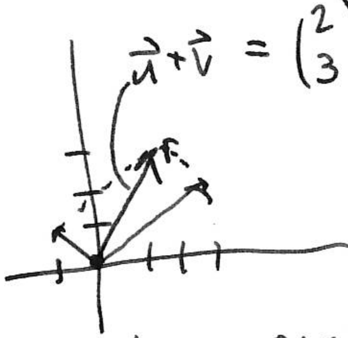
$\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ ← x coord, ← y coord, ← z coord.

$\vec{u} + \vec{v}$, geometrically

$\vec{u} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$

$\vec{v} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$

$\vec{u} + \vec{v} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$



draw a parallelogram

$c\,\vec{u}$, geometrically

c ← number, $\vec{u}$ ← vector

$\vec{u} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$

$2\vec{u} = \begin{pmatrix} 6 \\ 4 \end{pmatrix}$