Announcements

Q next

- Exam next W
- Quizzes will be returned M
- Pick up Quiz 8 on the way out today
- Tell me if you need a make-up!

- Quick review of complex numbers: Appendix B.

How to diagonalize a matrix A

$$A = \begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix}.$$

1. find the eigenvalues.

want: $\det(A - \lambda I) = 0$

$$\det\begin{pmatrix} 1-\lambda & 2 \\ 0 & 3-\lambda \end{pmatrix} = (1-\lambda)(3-\lambda) - 0 \cdot 2$$

$$= (1-\lambda)(3-\lambda).$$

roots are $\lambda = 1$, $\lambda = 3$.

2. Find an eigenvector for each eigenvalue.

for each λ, solve $(A - \lambda I)\vec{x} = \vec{0}$.

$\lambda = 1$: $A - \lambda I = \begin{pmatrix} 0 & 2 \\ 0 & 2 \end{pmatrix}$. $(A - \lambda I)\vec{x} = \vec{0} \rightsquigarrow \left[\begin{array}{cc|c} 0 & 2 & 0 \\ 0 & 2 & 0 \end{array}\right]$

row reduction.

$\vec{x} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$.

$\lambda = 3$, $A - \lambda I = \begin{pmatrix} -2 & 2 \\ 0 & 0 \end{pmatrix}$. $(A - \lambda I)\vec{x} = \vec{0}$.

$\vec{x} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

3. write down P & D:

evectrs

$$P = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$

eigenvals

$$D = \begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix}.$$

$$\boxed{A = PDP^{-1}}$$

why eigenbases useful?

for most matrices A, the eigenvectors give a basis for $\mathbb{R}^n$ (where A is $n \times n$)

that's an eigenbasis.

say we want to know $A^n \vec{x}$ for large n.

if $\vec{x}$ is a combination of eigenvectors, this is easy:

$$A = \begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix} \quad \vec{x} = \begin{bmatrix} 3 \\ 2 \end{bmatrix} = 1\begin{bmatrix} 1 \\ 0 \end{bmatrix} + 2\begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

$$A\vec{x} = 1 \cdot 1 \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix} + 2 \cdot 3 \cdot \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$A^2\vec{x} = 1 \cdot 1^2 \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix} + 2 \cdot 3^2 \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$A^3\vec{x} = 1 \cdot 1^3 \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix} + 2 \cdot 3^3 \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\ldots,$$

$$A^n\vec{x} = 1 \cdot 1^n \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix} + 2 \cdot 3^n \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

in the formulas from Monday,

$$[\vec{x}]_{\mathcal{B}} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad \left(\text{since } \vec{x} = 1 \overset{\vec{b}_1}{\begin{bmatrix} 1 \\ 0 \end{bmatrix}} + 2 \overset{\vec{b}_2}{\begin{bmatrix} 1 \\ 1 \end{bmatrix}}\right)$$

$$\cancel{A^n \vec{x} = \begin{bmatrix} 1^n & 0 \\ 0 & 3^n \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix}}$$

$$[A^n \vec{x}]_{\mathcal{B}} = \begin{bmatrix} 1^n & 0 \\ 0 & 3^n \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \cdot 1^n \\ 2 \cdot 3^n \end{bmatrix}$$

since $A^n \vec{x} = 1 \cdot 1^n \begin{bmatrix} 1 \\ 0 \end{bmatrix} + 2 \cdot 3^n \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

this shows:

$$\boxed{[A^n \vec{x}]_{\mathcal{B}} = D^n \, [\vec{x}]_{\mathcal{B}}.}$$

(key formula from Monday!)

looks scary, but it's just a compact way to write the observations from the first page.

Recap of complex numbers

What are eigenvalues of ~~aAA~~

$$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}? \quad \text{(rotation by } 90^\circ\text{)}$$

$$\det(A-\lambda I) = \det\begin{pmatrix} -\lambda & -1 \\ 1 & -\lambda \end{pmatrix} = \lambda^2 + 1.$$

so $\lambda^2 + 1 = 0$.

uhoh, - need complex numbers: $\lambda = i$ or $-i$.

make sure you know how to:

- add complex #s
- multiply them
- divide
- work in polar coords: $a + bi = re^{i\theta}$.

(this is explained well in Appendix B)

(or many many places online; email me to set up a time to go over it.)