Math 310, Lesieutre
Quiz #1
September 2, 2015

1. *Which of these matrices are in echelon form? For each matrix, show the next row operation you would use to reach reduced echelon form. (You only need to show one row operation, and it won't necessarily reach rref.)*

   (a)

   $$\left[\begin{array}{ccc|c} 2 & 4 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

   This is in echelon form. The next move is to divide the first row by 2:

   $$\left[\begin{array}{ccc|c} 2 & 4 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 2 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

   (b)

   $$\left[\begin{array}{cc|c} 1 & 0 & 2 \\ 1 & 3 & 7 \end{array}\right]$$

   This is not in echelon form. According to the algorithm, we should add -1 times the first row to the second row.

   $$\left[\begin{array}{cc|c} 1 & 0 & 2 \\ 1 & 3 & 7 \end{array}\right] \to \left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 3 & 5 \end{array}\right]$$

   (c)

   $$\left[\begin{array}{cc|c} 0 & 1 & 3 \\ 1 & 0 & 4 \end{array}\right]$$

   This is also not in echelon form. We should switch the two rows.

   $$\left[\begin{array}{cc|c} 0 & 1 & 3 \\ 1 & 0 & 4 \end{array}\right] \to \left[\begin{array}{cc|c} 1 & 0 & 4 \\ 0 & 1 & 3 \end{array}\right]$$

2. *Find the general solution to the following linear system by using row reduction on the augmented matrix:*

   $$\begin{aligned} x_1 - 2x_2 + 2x_3 &= 5 \\ -2x_1 + 4x_2 + 3x_3 &= 4 \end{aligned}$$

   How many solutions are there?

Start by writing the matrix and doing row reduction:

$$\begin{bmatrix} 1 & -2 & 2 & 5 \\ -2 & 4 & 3 & 4 \end{bmatrix} \to \begin{bmatrix} 1 & -2 & 2 & 5 \\ 0 & 0 & 7 & 14 \end{bmatrix} \to \begin{bmatrix} 1 & -2 & 2 & 5 \\ 0 & 0 & 1 & 2 \end{bmatrix} \to \begin{bmatrix} 1 & -2 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

The basic variables (the ones in the pivot columns) are x_1 and x_3. The variable x_2 is free. So the general solution is:

$$\begin{cases} x_1 = 1 + 2x_2, \\ x_2 \text{ is free}, \\ x_3 = 2. \end{cases}$$

Since the system is consistent and there is a free variable, there are infinitely many solutions.