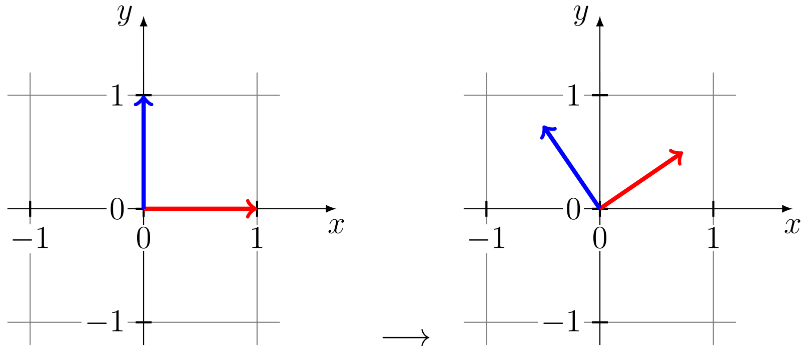
Math 310, Lesieutre
Quiz #4
September 23, 2015

Name: ______________________________

1. *Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation that rotates points (about the origin) 30° counterclockwise.*

   (a) *What is the matrix for T?*



Remember the strategy to figure out the matrix for a transformation: you need to look at the two vectors $\mathbf{e}_1 = (1, 0)$ and $\mathbf{e}_2 = (0, 1)$, and figure out what the transformation does to these two. In this case, a little geometry shows us that the coordinates for the rotated vectors on the right are

$$T(\mathbf{e}_1) = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right), \quad T(\mathbf{e}_2) = \left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$$

To get the matrix for T, we need to take $T(\mathbf{e}_1)$ and insert it as the first column of the matrix, and take $T(\mathbf{e}_2)$ and insert it as the second column:

$$A = \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$$

   (b) *Is the linear transformation T one-to-one? Why or why not?*

Remember that T is one-o-one if every $\mathbf{b}$ in $\mathbb{R}^2$ is the image of *at most one* $\mathbf{x}$. This is the case for this transformation: given any vector $\mathbf{b}$, there is never more than one vector that gets sent to $\mathbf{b}$, since the map is just a rotation.

You could also check this using row reduction on the matrix you found in (a): you need to verify that the columns of A are linearly independent.

2. *Consider the matrices:*

$$A = \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 3 & 1 \\ 2 & 2 & 1 \end{bmatrix}.$$

For each of AB and BA, either compute it or explain why it is not defined.

$$AB = \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 & 3 & 1 \\ 2 & 2 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 1 & 04 & 4 & 2 \end{bmatrix}$$

$$BA = \begin{bmatrix} 1 & 3 & 1 \\ 2 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} = \text{Not defined: the numbers of rows of the}$$

second matrix is not equal to the number of columns of the first one.