Math 310, Lesieutre
Quiz #5
October 7, 2015

Name: ______________________________

1. *Consider the matrix*

$$A = \begin{bmatrix} 1 & -1 \\ -2 & 4 \\ -1 & 2 \end{bmatrix}.$$

Compute an LU decomposition for A.

We need to do row reduction. That goes:

$$\begin{bmatrix} 1 & -1 \\ -2 & 4 \\ -1 & 2 \end{bmatrix} \to \begin{bmatrix} 1 & -1 \\ 0 & 0 \\ -1 & 2 \end{bmatrix} \to \begin{bmatrix} 1 & -1 \\ 0 & 2 \\ 0 & 1 \end{bmatrix} \to \begin{bmatrix} 1 & -1 \\ 0 & 2 \\ 0 & 0 \end{bmatrix}.$$

The reductions were:

(a) Subtract (-2) row 1 from row 2,
(b) Subtract (-1) row 1 from row 3,
(c) Subtract $1/2$ row 2 from row 3.

The matrix U we just get from the echelon form we ended up with. It's the same size as A.

$$U = \begin{bmatrix} 1 & -1 \\ 0 & 2 \\ 0 & 0 \end{bmatrix}$$

The matrix L should be 3×3. It gets 1s on the diagonal and 0s above it, and the entries below the diagonal come from which row reductions we used.

$$L = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -1 & \frac{1}{2} & 1 \end{bmatrix}.$$

You can always double-check your LU decomposition: just multiply out LU, and you had better get the A you started with.

$$LU = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -1 & \frac{1}{2} & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 0 & 2 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -2 & 4 \\ -1 & 2 \end{bmatrix} = A.$$

Phew, it worked.

2. (a) *Compute the determinant of the following matrix. You can use any strategy or combination of strategies that you think is appropriate.*

$$A = \begin{bmatrix} 0 & 0 & 2 & 3 \\ 0 & 0 & -1 & -2 \\ 1 & 2 & 3 & 4 \\ 1 & 5 & 1 & 2 \end{bmatrix}$$

There are a bunch of ways to do this. The easiest (and most popular on the quiz) is probably cofactor expansion down the first column. I'll do it by row reduction just so you can see how it goes.

First, swap row 1 and row 3, and swap row 2 and row 4.

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 5 & 1 & 2 \\ 0 & 0 & 2 & 3 \\ 0 & 0 & -1 & -2 \end{bmatrix}$$

Now row reduce: add -1 times row 1 to row 2, and add $1/2$ times row 3 to row 4. Neither of these things changes the determinant.

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 5 & 1 & 2 \\ 0 & 0 & 2 & 3 \\ 0 & 0 & -1 & -2 \end{bmatrix} \to \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 3 & -2 & -2 \\ 0 & 0 & 2 & 3 \\ 0 & 0 & 0 & -1/2 \end{bmatrix}$$

This guy is upper triangular, so we know the determinant: it's $(1)(3)(2)(-1/2) = -3$. The determinant of the original is therefore $(-1)^{\#\text{ of row swaps}}(-3) = (-1)^2(-3) = -3$.

(b) *What is the determinant of A^3?*

We know $\det(A^3) = (\det A)^3 = (-3)^3 = -27$. Note that this is a lot less work than computing A^3 and finding the determinant of that, though you'd get the same answer.