Math 310, Lesieutre
Problem set #12
November 18, 2015

Problems for M 11/9:

5.7.1 A particle moving in a force field has a position vector $\mathbf{x}$ that satisfies $\mathbf{x}' = A\mathbf{x}$. The 2×2 matrix A has eigenvalues 4 and 2, with corresponding eigenvectors $\mathbf{v}_1 = \begin{bmatrix} -3 \\ 1 \end{bmatrix}$ and $\mathbf{v}_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$. Find the position of the particle at time t, assuming that $\mathbf{x}(0) = \begin{bmatrix} -6 \\ 1 \end{bmatrix}$.

5.7.7 Find a change of variable that decouples the equation $\mathbf{x}' = A\mathbf{x}$. Write the equation $\mathbf{x}(t) = P\mathbf{y}(t)$ and show the calculation that leads to the uncoupled system $\mathbf{y}' = D\mathbf{y}$, specifying P and D.

$$A = \begin{bmatrix} 7 & -1 \\ 3 & 3 \end{bmatrix}$$

5.7.9 Construct the general solution of $\mathbf{x}' = A\mathbf{x}$ involving complex eigenfunctions and then obtain the general real solution.

$$A = \begin{bmatrix} -3 & 2 \\ -1 & -1 \end{bmatrix}$$

Problems for F 11/13:

6.1.1 Let $\mathbf{u} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} 4 \\ 6 \end{bmatrix}$. Compute $\mathbf{u}\cdot\mathbf{u}$, $\mathbf{u}\cdot\mathbf{v}$, and $\frac{\mathbf{v}\cdot\mathbf{u}}{\mathbf{u}\cdot\mathbf{u}}$.

6.1.14 Find the distance between the two vectors

$$\begin{bmatrix} 0 \\ -5 \\ 2 \end{bmatrix}, \quad \begin{bmatrix} -4 \\ -1 \\ 8 \end{bmatrix}.$$

6.1.16 Are the following two vectors orthogonal?

$$\begin{bmatrix} 12 \\ 3 \\ -5 \end{bmatrix}, \quad \begin{bmatrix} 2 \\ -3 \\ 3 \end{bmatrix}.$$

6.1.26 Let $\mathbf{u} = \begin{bmatrix} 5 \\ -6 \\ 7 \end{bmatrix}$, and let W be the set of all vectors $\mathbf{x}$ with $\mathbf{u}\cdot\mathbf{x} = 0$. Explain why W is a subspace of $\mathbb{R}^3$, and describe it geometrically.