Math 310, Lesieutre
Problem set #5
September 30, 2015

Problems for M 9/21:

2.2.1 Find the inverse of

$$\begin{bmatrix} 8 & 6 \\ 5 & 4 \end{bmatrix}.$$

2.2.5 Use the inverse from the first problem to solve the system

$$\begin{aligned} 8x_1 + 6x_2 &= 2 \\ 5x_1 + 4x_2 &= -1 \end{aligned}$$

2.2.31 Find the inverse of the following matrix, using the row reduction method discussed in class:

$$A = \begin{bmatrix} 1 & 0 & -2 \\ -3 & 1 & 4 \\ 2 & -3 & 4 \end{bmatrix}$$

#4 Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation defined by rotating a vector 45 degrees clockwise. Last week, you found the matrix for T: call it A.

(a) Compute A^{-1}.

(b) Compute $A^{-1}\mathbf{v}$ for a couple vectors of your choice. How does A^{-1} transform vectors?

Problems for W 9/23:

2.3.1 Determine if the following matrix is invertible, using the method of your choice. Try to use as few calculations as possible:

$$A = \begin{bmatrix} 5 & 7 \\ -3 & -6 \end{bmatrix}$$

2.3.3 Same problem, with:

$$A = \begin{bmatrix} 5 & 0 & 0 \\ -3 & -7 & 0 \\ 8 & 5 & -1 \end{bmatrix}$$

2.3.13 An $m \times n$ *upper triangular matrix* is one whose entries below the main diagonal are all 0. When is a square upper triangular matrix invertible? Justify your answer.

2.3.15 Can a square matrix with two identical columns be invertible? Why or why not?

Problems for F 9/25:

1.10.9 In a certain region, about 7% of a city's population moves to the usrrounding suburbs each year, and about 5% of the suburban population moves into the city. In 2015, there were 800,000 residents in the city and 500,000 in the suburbs. Set up a difference equation that describes this situation, where $\mathbf{x}_0$ is the initial population in 2015. Then estimate the populations in the city and in the suburbs two years later, in 2017. (This one is a repeat from last week, but now we actually covered it in lecture.)

2.5.1 Solve $A\mathbf{x} =$ using the LU decomposition:

$$A = \begin{bmatrix} 3 & -7 & -2 \\ -3 & 5 & 1 \\ 6 & -4 & 0 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} -7 \\ 5 \\ 2 \end{bmatrix}.$$

2.5.9 Find an LU decomposition for the following matrix:

$$A = \begin{bmatrix} 3 & -1 & 2 \\ -3 & -2 & 10 \\ 9 & -5 & 6 \end{bmatrix}.$$