Math 310, Lesieutre
Problem set #7
October 14, 2015

Problems for M 10/5:

3.3.1 Use Cramer's rule to solve $5x_1 + 7x_2 = 3$, $2x_1 + 4x_2 = 1$.

3.3.7 Determine the values of s for which the system has a unique solution, and describe the solution:

$$\begin{aligned} 6sx_1 + 4x_2 &= 5 \\ 9x_1 + 2sx_2 &= -2. \end{aligned}$$

3.3.11 Find the adjugate of the given matrix and use it to find the inverse:

$$A = \begin{bmatrix} 0 & -2 & -1 \\ 5 & 0 & 0 \\ -1 & 1 & 1 \end{bmatrix}$$

(We didn't define the adjugate in class, though we got close: take a look at page 181.)

3.3.20 Find the area of the parallelogram with vertices $(0,0)$, $(2,-4)$, $(4,-5)$, and $(2,-1)$

3.3.29 Find a formula for the area of the triangle whose vertices are $\mathbf{0}$, $\mathbf{v}_1 = (a,b)$, and $\mathbf{v}_2 = (c,d)$ in $\mathbb{R}^2$.

Problems for W 10/7:

4.1.1 Let V be the first quadrant in the xy-plane, that is the set of all vectors (x,y) with $x \geq 0$ and $y \geq 0$. In set notation, this is:

$$V = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} : x \geq 0, y \geq 0 \right\}.$$

(a) If $\mathbf{u}$ and $\mathbf{v}$ are in V, is $\mathbf{u} + \mathbf{v}$ in V?

(b) Find a specific vector $\mathbf{u}$ in V and a specific scalar c such that $c\mathbf{u}$ is not in V. (This is enough to show that V is not a vector space.)

4.1.3 Let H be the set of points inside and on the unit circle in the xy-plane:

$$H = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} : x^2 + y^2 \leq 1 \right\}.$$

Find a specific example – two vectors or a vector and a scalar – to show that H is not a subspace of $\mathbb{R}^2$.

4.1.5 Let $\mathbb{P}_2$ be the vector space of polynomials of degree at most 2. Is the set of polynomials of the form at^2 a subset of $\mathbb{P}_2$ (where a is a scalar?)

4.1.7 Let $\mathbb{P}_2$ be the vector space of polynomials of degree at most 3. Is the set of all polynomials with integers as coefficients a subspace?

Problems for F 10/9:

4.1.11 Let W be the set of all vectors of the form $\begin{bmatrix} 5b+2c \\ b \\ c \end{bmatrix}$. Find vectors $\mathbf{u}$ and $\mathbf{v}$ such that $W = \text{span}(\mathbf{u}, \mathbf{v})$. Why does this show that W is a subspace of $\mathbb{R}^n$?

4.2.4 Find an explicit description of the nullspace of the following matrix by listing a set of vectors that span the nullspace:

$$A = \begin{bmatrix} 1 & -6 & 4 & 0 \\ 0 & 0 & 2 & 0 \end{bmatrix}$$

4.2.7 Explain why the following set either is or is not a vector space:

$$\left\{ \begin{bmatrix} a \\ b \\ c \end{bmatrix} : a+b+c=2 \right\}$$

4.2.15 Find a matrix A such that the given set is the column space of A.

$$\left\{ \begin{bmatrix} 2s+3t \\ r+s-2t \\ 4r+s \\ 3r-s-t \end{bmatrix} : r, s, t \text{ are scalars} \right\}.$$

4.2.17 For what value of k is $\text{Nul}(A)$ a subspace of $\mathbb{R}^k$? For what value of k is $\text{Col}(A)$ a subspace of $\mathbb{R}^k$?

$$A = \begin{bmatrix} 2 & -6 \\ -1 & 3 \\ -4 & 12 \\ 3 & -9 \end{bmatrix}.$$