Math 310, Lesieutre
Problem set #8
October 21, 2015

Problems for M 10/12:

4.3.3 Determine whether these vectors are a basis for $\mathbb{R}^3$ by checking whether the vectors span $\mathbb{R}^3$, and whether the vectors are linearly independent.

$$\begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}, \quad \begin{bmatrix} 3 \\ 2 \\ -4 \end{bmatrix}, \quad \begin{bmatrix} -3 \\ -5 \\ 1 \end{bmatrix}.$$

4.3.9 Find a basis for the nullspace of the following matrix:

$$\begin{bmatrix} 1 & 0 & -3 & 2 \\ 0 & 1 & -5 & 4 \\ 3 & -2 & 1 & -2 \end{bmatrix}.$$

4.3.15 Find a basis for the space spanned by the vectors below:

$$\begin{bmatrix} 1 \\ 0 \\ -3 \\ 2 \end{bmatrix}, \quad \begin{bmatrix} 0 \\ 1 \\ 2 \\ -3 \end{bmatrix}, \quad \begin{bmatrix} -3 \\ -4 \\ 1 \\ 6 \end{bmatrix}, \quad \begin{bmatrix} 1 \\ -3 \\ -8 \\ 7 \end{bmatrix}, \quad \begin{bmatrix} 2 \\ 1 \\ -6 \\ 9 \end{bmatrix}.$$

4.3.34 Consider the polynomials $\mathbf{p}_1(t) = 1 + t$, $\mathbf{p}_2(t) = 1 - t$, and $\mathbf{p}_3(t) = 2$. Write down a linear dependence among these three polynomials. Find a basis for the span of these three polynomials.

Problems for W 10/14:

4.4.3 Find the vector $\mathbf{x}$ determined by the given coordinate vector $[\mathbf{x}]_{\mathcal{B}}$ and the given basis $\mathcal{B}$:

$$\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix}, \begin{bmatrix} 5 \\ 2 \\ -2 \end{bmatrix}, \begin{bmatrix} 4 \\ -7 \\ 0 \end{bmatrix} \right\}, \qquad [\mathbf{x}]_{\mathcal{B}} = \begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix}.$$

4.4.5 Find the coordinate vector $[\mathbf{x}]_{\mathcal{B}}$ of $\mathbf{x}$ relative to the given basis $\mathcal{B}$:

$$\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ -3 \end{bmatrix}, \begin{bmatrix} 2 \\ -5 \end{bmatrix} \right\}, \qquad \mathbf{x} = \begin{bmatrix} -2 \\ 1 \end{bmatrix}.$$

4.4.7

$$\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ -1 \\ -3 \end{bmatrix}, \begin{bmatrix} -3 \\ 4 \\ 9 \end{bmatrix}, \begin{bmatrix} 2 \\ -2 \\ 4 \end{bmatrix} \right\}, \qquad \mathbf{x} = \begin{bmatrix} 8 \\ -9 \\ 6 \end{bmatrix}.$$

4.4.12 Use an inverse matrix to find $[\mathbf{x}]_{\mathcal{B}}$ for the given $\mathbf{x}$ and $\mathcal{B}$.

$$\mathcal{B} = \left\{ \begin{bmatrix} 4 \\ 5 \end{bmatrix}, \begin{bmatrix} 6 \\ 7 \end{bmatrix} \right\}, \qquad \mathbf{x} = \begin{bmatrix} 2 \\ -6 \end{bmatrix}.$$

4.4.14 The set $\mathcal{B} = \{1 - t^2, t - t^2, 2 - 2t + t^2\}$ is a basis for $\mathbb{P}_2$. Find the coordinate vector of $\mathbf{p}(t) = 3 + t - 6t^2$. relative to $\mathcal{B}$.

Problems for F 10/16:

4.5.1 For the following subspace, find a basis, and state the dimension:

$$\left\{ \begin{bmatrix} s - 2t \\ s + t \\ 3t \end{bmatrix} : s, t \in \mathbb{R} \right\}.$$

4.5.5

$$\left\{ \begin{bmatrix} a - 4b - 2c \\ 2a + 5b - 4c \\ -a + 2c \\ -3a + 7b + 6c \end{bmatrix} : s, t \in \mathbb{R} \right\}.$$

4.5.11 Find the dimension of the subspace spanned by the following vectors:

$$\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 9 \\ 4 \\ -2 \end{bmatrix}, \begin{bmatrix} -7 \\ -3 \\ 1 \end{bmatrix}.$$

4.5.21 The first four Hermite polynomials are 1, $2t$, $2 - 4t + t^2$, and $6 - 18t + 9t^2 - t^3$. Show that these polynomials form a basis for $\mathbb{P}_3$.