Math 310, Lesieutre
Problem set #9
October 28, 2015

Problems for M 10/19:

4.6.1 Row reduction on the matrix A below yields the matrix B. Without calculations, list $\operatorname{rank} A$ and $\dim \operatorname{Nul} A$. Find bases for $\operatorname{Col} A$, $\operatorname{Row} A$, and $\operatorname{Nul} A$.

$$A = \begin{bmatrix} 1 & -4 & 9 & -7 \\ -1 & 2 & -4 & 1 \\ 5 & -6 & 10 & 7 \end{bmatrix}$$
$$B = \begin{bmatrix} 1 & 0 & -1 & 5 \\ 0 & -2 & 5 & -6 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

4.6.2 Same deal.

$$A = \begin{bmatrix} 1 & -3 & 4 & -1 & 9 \\ -2 & 6 & -6 & -1 & -10 \\ -3 & 9 & -6 & -6 & -3 \\ 3 & -9 & 4 & 9 & 0 \end{bmatrix}$$
$$B = \begin{bmatrix} 1 & -3 & 0 & 5 & -7 \\ 0 & 0 & 2 & -3 & 8 \\ 0 & 0 & 0 & 0 & 5 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

4.6.8 Suppose that a 5×6 matrix A has four pivot columns. What is $\dim \operatorname{Nul} A$? Is $\operatorname{Col} A = \mathbb{R}^4$? Why or why not?

4.6.9 If the null space of a 5×6 matrix A is 4-dimensional, what is the dimension of the column space of A?

4.6.22 Is it possible that all solutions of a homogeneous sytem of ten linear equations in twelve variables are multiples of one fixed nonzero solution? Discuss.

Problems for W 10/21:

4.7.1 Let $\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2\}$ and $\mathcal{C} = \{\mathbf{c}_1, \mathbf{c}_2\}$ be basis for a vector space V, and suppose that $\mathbf{b}_1 = 6\mathbf{c}_1 - 2\mathbf{c}_2$ and $\mathbf{b}_2 = 9\mathbf{c}_1 - 4\mathbf{c}_2$.

(a) Find the change-of-coordinates matrix from $\mathcal{B}$ to $\mathcal{C}$ (i.e. the matrix $\underset{\mathcal{C}\leftarrow\mathcal{B}}{\mathcal{P}}$).

(b) Find $[\mathbf{x}]_{\mathcal{C}}$ for $\mathbf{x} = -3\mathbf{b}_1 + 2\mathbf{b}_2$. (Use part (a).)

4.7.7 Let $\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2\}$ and $\mathcal{C} = \{\mathbf{c}_1, \mathbf{c}_2\}$ be the bases for $\mathbb{R}^2$ listed below.

$$\mathbf{b}_1 = \begin{bmatrix} 7 \\ 5 \end{bmatrix}, \quad \mathbf{b}_2 = \begin{bmatrix} -3 \\ -1 \end{bmatrix}, \quad \mathbf{c}_1 = \begin{bmatrix} 1 \\ -5 \end{bmatrix}, \quad \mathbf{c}_1 = \begin{bmatrix} -2 \\ 2 \end{bmatrix}.$$

Find the change-of-coordinates matrix from $\mathcal{B}$ to $\mathcal{C}$ and from $\mathcal{C}$ to $\mathcal{B}$.

4.7.9 Again, with

$$\mathbf{b}_1 = \begin{bmatrix} -6 \\ -1 \end{bmatrix}, \quad \mathbf{b}_2 = \begin{bmatrix} 2 \\ 0 \end{bmatrix}, \quad \mathbf{c}_1 = \begin{bmatrix} 2 \\ -1 \end{bmatrix}, \quad \mathbf{c}_1 = \begin{bmatrix} 6 \\ -2 \end{bmatrix}.$$

4.7.13 In $\mathbb{P}_2$, find the change-of-coordinates matrix from the basis

$$\mathcal{B} = \left\{1 - 2t + t^2, 3 - 5t + 4t^2, 2t + 3t^2\right\}$$

to the standard basis $\mathcal{C} = \{1, t, t^2\}$. Then find the $\mathcal{B}$-coordinate vector for $-1 + 2t$.

Problems for F 10/23:

4.9.1 A small remote village receives radio broadcasts from two radio stationts, a news station and a music station. Of the listeners who are tuned to the news station, 70% will remain listening to the news after the break that occurs each half hour, while 30% will switch to the music station at the station break. Of the listeners who are tuned to the music station, 60% will switch at the break, while 40% will remain. Suppose that at 8:15 AM, everyone is listening to the news.

(a) Give the stochastic matrix that describes how the radio listeners tend to change stations at ecah station break. Label the rows and columns.

(b) Give the initial state vector.

(c) What percentage of the listeners will be listening to the music station at 9:25 AM, after breaks at 8:30 and 9:00 AM?

4.9.4 The weather in Columbus is either good, indifferent, or bad on any given day. If the weather is good today, there is a 60% chance that the weather will be good tomorrow, 30% that it's indifferent, and 10% that it's bad. If it's indifferent today, it's good tomorrow with probability 40%, indifferent 30%. If bad today, it's good with probability 40% and indifferent 50%.

(a) What is the stochastic matrix for this situation?

(b) Suppose there is a 50% chance of good weather today and 50% of indifferent. What are the chances of bad weather tomorrow?

(c) Suppose the predicted weather for monday is 40% indifferent and 60% bad. What are the chances for good weather on wednesday?

4.9.7 Find the steady-state vector for the following stochastic matrix:

$$a = \begin{bmatrix} 0.7 & 0.1 & 0.1 \\ 0.2 & 0.8 & 0.2 \\ 0.1 & 0.1 & 0.7 \end{bmatrix}.$$

4.9.11 Find the steady state for the markov chain in exercise 4.9.1 (the radio thing). At some time late in the day, what fraction of the listeners will be listening to the news?