Let's try to construct an ample line bundle on the blow-up of $\mathbb{P}^2$ at 2 points, and prove it's ample using the Nakai criterion.

I claim that $3H - E_1 - E_2$, the line bundle associated to a cubic through the two points, is going to do the trick. There are two things we need to know: first, that it has positive degree when restricted to the cubic itself, and second, that it has positive degree on every curve in the surface.

We know that $3H$ restricted to the cubic has degree 9. Similarly, E_i restricted to the cubic has degree 1. Since the E_i appear with negative signs, the degree is $9 - 1 - 1 = 7$, which is positive, as needed.

Now we need to show that given any curve on X, the cubic in question has positive degree on the curve. This is certainly true for the two curves E_1 and E_2, where the degree is 1. Every other curve on X has image a curve in $\mathbb{P}^2$ of some degree d and some multiplicities m_1 and m_2 at the two poins in question.

What's the degree of $3H - E_1 - E_2$ restricted to such a curve on X? Well, $E_i|_C$ has degree m_i: these two things intersect on X at m_i points. And $3H|_C$ has degree $3d$. So the degree of the restriction is $3d - m_1 - m_2$. On the other hand, we know that $d \geq m_1$ and $d \geq m_2$, since a curve can't have multiplicity at a point that's larger than its degree. So $\deg(3H - E_1 - E_2)|_C = 3d - m_1 - m_2 > 0$, and the line bundle is ample according to the Nakai criterion.