

Math 553, Lesieutre
Problem set #1
November 4, 2015

1. II.1.2

2. II.1.4

3. II.1.5

4. II.1.8

5. Let \mathcal{F} be the presheaf on \mathbb{C} (with the usual topology) defined by

$$\mathcal{F}(U) = \{f \in \mathcal{O}_{\mathbb{C}}(U) : f = \exp(g) \text{ for some } g \in \mathcal{O}_{\mathbb{C}}(U)\}.$$

We saw that this fits into an exact sequence of presheaves

$$0 \longrightarrow 2\pi i\mathbb{Z} \longrightarrow \mathcal{O}_{\mathbb{C}} \xrightarrow{\exp} \mathcal{F} \longrightarrow 1$$

Explain why applying the construction of the sheafification to \mathcal{F} gives the sheaf of nonvanishing functions $\mathcal{O}_{\mathbb{C}}^*$. Check that

$$0 \longrightarrow 2\pi i\mathbb{Z} \longrightarrow \mathcal{O}_{\mathbb{C}} \xrightarrow{\exp} \mathcal{O}_{\mathbb{C}}^* \longrightarrow 1$$

is an exact sequence of sheaves.

6. II.1.14

7. II.1.21a,b

8. Suppose that \mathcal{F} is a sheaf on X , and let $f : p \rightarrow X$ be the inclusion of a point $p \in X$. Describe the sheaf $f^{-1}(\mathcal{F})$.