Math 553, Lesieutre
Problem set #11
due April 10, 2016

1. Compute $H^1(S^1, \mathbb{Z})$ (the derived functor cohomology), where S^1 is the circle and $\mathbb{Z}$ is the constant sheaf.

2. Suppose that X is a smooth, compact manifold. Show that the de Rham complex

$$0 \to \underline{\mathbb{R}} \to C^\infty(X) \to \Lambda^1(X) \to \Lambda^2(X) \to \cdots$$

   provides an acyclic resolution. (Hint: use partitions of unity.)

3. Let $X = \operatorname{Spec} A$ be an affine scheme. Suppose that $\mathcal{F}$ is an a quasicoherent sheaf on X, and that $\mathcal{U}$ is an affine cover of X. Show that the Čech cohomology groups $\check{H}^i(X, \mathcal{U}, \mathcal{F})$ vanish for $i > 0$. (You can look in Vakil's book if you need help getting started.)

4. III.4.1

5. III.4.3

6. Suppose that X is a smooth variety over $\mathbb{C}$, D is a smooth divisor on X, and L is an invertible sheaf on X. Often we'd like to "lift sections" of L from D, i.e. show that some $s \in H^0(D, L|_D)$ is in the image of $H^0(X, L) \to H^0(D, L|_D)$.

   The vanishing of some cohomology group of some sheaf implies that the map in question is actually surjective. What vanishing do we need?

   Give an example of X, D, L as above where the map in question is not surjective.