Math 553, Lesieutre
Problem set #2
due January 27, 2016

1. Consider the ideal $I = (xy, y^2) \subset \mathbb{C}[x, y]$ and the scheme $X = \operatorname{Spec} \mathbb{C}[x, y]/I)$. There is a map $X \to \mathbb{A}^2_{\mathbb{C}}$ induced by the quotient map $\mathbb{C}[x, y] \to \mathbb{C}[x, y]/I$. Given a function $f \in \Gamma(\operatorname{Spec} \mathbb{C}[x], \mathcal{O})$, we get a function $f_0 \in \Gamma(\operatorname{Spec} X, \mathcal{O}_X)$. What information about f does f_0 remember?

2. Describe the topological space underlying the scheme X. Describe the stalks of the structure sheaf at each point.

3. Describe all the data involved in the map of schemes $f : X \to \mathbb{A}^2_{\mathbb{C}}$ (e.g., tell me the map on spaces and describe the map of sheaves $f^\sharp : \mathcal{O}_{\mathbb{A}^2_{\mathbb{C}}} \to f_* \mathcal{O}_X$).

4. II.2.2

5. II.2.3

6. II.2.7

7. II.2.8

8. Optional: rewrite the proof of Proposition 2.2(b) from Hartshorne. This is the fact that $\mathcal{O}(D(f))$ is isomorphic to A_f, and it's a good exercise in understanding the definition of the structure sheaf as the book defines it.