Math 553, Lesieutre
Problem set #6
due February 26, 2016

1. Compute the global sections $\Gamma(\mathbb{P}^n_k, \mathcal{O}(d))$. (You can extract this from Prop. 5.13.)

2. Show that the invertible sheaf $\mathcal{O}_{\mathbb{P}^1_k}(2)$ is very ample by giving a map $\mathbb{P}^1_k \to \mathbb{P}^2_k$ with $i^*(\mathcal{O}_{\mathbb{P}^2_k}(1)) \cong \mathcal{O}_{\mathbb{P}^1_k}(2)$.

3. Let x and y be two closed points in $\mathbb{P}^2_{\mathbb{C}}$.

   (a) Let $d \geq 1$ and $m_1, m_2 \geq 0$ be integers. Describe the coherent sheaf $\mathcal{F}_{d,m_1,m_2} = \mathcal{O}_{\mathbb{P}^2}(d) \otimes \mathcal{I}_x^{\otimes m_1} \otimes \mathcal{I}_y^{\otimes m_2}$. What are the global sections?

   (b) Let $d = 1$ and $m_1 = m_2 = 1$. Compute $\Gamma(\mathbb{P}^2, \mathcal{F}_{1,1,1})$. Is this sheaf globally generated?

   (c) The sheaf $\mathcal{F}_{1,0,0} = \mathcal{O}(1)$ is very ample on $\mathbb{P}^2$, so we know that $\mathcal{F}_{1,1,1} \otimes \mathcal{O}(1)^{\otimes k}$ is globally generated for some k. What k is needed?

4. 5.11

5. 5.12

6. Read through the statements all of the following problems, and solve at least three parts of one of them: 5.15, 5.16, 5.18.